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<b>14. ABSTRACT</b> The central goal of this project was the investigation of the enhancements in antenna performance and in imaging capability by embedding a source or scatterer in a background medium including metamaterials. This effort was motivated mostly by the possibility of embedding antennas in designer background media so as to obtain radiation performance that would not be possible for comparable antennas in free space. This problem was treated in the present work within a general and non-device-specific framework whose predictions (such as performance bounds) under normalized resources are fundamental. The results were discussed addressing separately the cases of small versus large or resonant antennas, with the overall conclusion that for small antennas one can significantly enhance the radiated power or compress source size via antenna substrates under normalized antenna resources, while for larger antennas the use of substrates can significantly enhance both radiated power and directivity (related to the number of essentially independent field modes that can be radiated effectively) under the given resources.					
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## **Final Report for AFOSR Grant No. FA9550-06-1-0013**

### **Inverse Source Problem in Non-homogeneous and Metamaterial Background Media: Antenna Synthesis and Performance Bounds**

Final Report Submitted to AFOSR by:

Dr. Edwin A. Marengo (Principal Investigator)  
Department of Electrical and Computer Engineering  
Northeastern University  
Boston, Massachusetts 02115

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## **I. Executive Summary**

This final report summarizes our work entitled "Inverse Source Problem in Non-homogeneous and Metamaterial Background Media: Antenna Synthesis and Performance Bounds", AFOSR Grant Number FA9550-06-1-0013, carried out under the support of the program of Dr. Arje Nachman at the AFOSR during the period Dec.1, 2005-Nov.30, 2008. The Principal Investigator (PI) for this effort was Dr. Edwin Marengo at Northeastern University. Part of the work was done in collaboration with Dr. Anthony Devaney, also at Northeastern University.

The central goal of this project was the investigation, via the formalism of the inverse source problem, of the enhancements in antenna performance

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and in imaging capability by embedding a source or scatterer in a background medium including metamaterials. This effort was motivated mostly by the possibility of embedding antennas in designer background media so as to obtain radiation performance that would not be possible for comparable antennas in free space.

This problem was treated in the present work within a general and non-device-specific framework based on the inverse source problem whose predictions (such as performance bounds) under normalized resources are fundamental. The results were discussed addressing separately the cases of small versus large or resonant antennas, with the overall conclusion that for small antennas one can significantly enhance the radiated power or compress source size via the antenna substrates under normalized antenna resources, while for larger antennas the use of substrates can significantly enhance both radiated power and directivity (related to the number of essentially independent field modes that can be radiated effectively) under the given resources.

The main accomplishments of this work are:

- The development of a new theory of the full vector electromagnetic inverse source problem in non-homogeneous and metamaterial background media, including solution constraints relevant to antenna synthesis (source functional energy or L2 norm, plus reactive power tuning).
- The demonstration, via many computer simulations, of the effectiveness of this theory as a tool for antenna analysis, characterization, and design, which enables the fair characterization and comparison of the performance of different antenna substrates.

## II. Personnel

### II.A. Supported Personnel

The following faculty were supported by this grant:

- Dr. Edwin A. Marengo (the PI of this work)  
Department of Electrical and Computer Engineering  
Northeastern University  
Boston, MA 02115

- Dr. Anthony J. Devaney (collaborator in this work)  
Department of Electrical and Computer Engineering  
Northeastern University  
Boston, MA 02115

The following graduate students at the PI's research lab at Northeastern University were supported by this grant:

- Fred Gruber
- Mohamed Khodja
- Ronald Hernandez

## II.B. Other Collaborators

The following are collaborators who were *not* supported by this grant, but with whom the PI worked regularly during the past three years on scientific problems connected to the grant:

- Dr. Francesco Simonetti  
Department of Mechanical Engineering  
Imperial College London  
London, SW7 2AZ, United Kingdom

**Description:** The collaboration with Dr. Francesco Simonetti at Imperial College in London was in connection with the exploration of the role of multiple scattering in imaging enhancement (see the paper [1]) and in signal-subspace-based imaging for extended scatterers (see the paper [2]).

- Dr. Johan Sten  
Technical Research Centre of Finland  
VTT Information Technology  
P.O. Box 1000  
FI-02044 VTT, Finland

**Description:** The collaboration with Dr. Johan Sten at VTT Finland and Helsinki University of Technology was on the study of the role of source shape in radiation (see the papers [3–5]).

There are a few other colleagues with whom the PI collaborated more sporadically. Thus we also collaborated with Dr. Gerald Kaiser of the Center for Signals and Waves in Austin, Texas, and Dr. Richard Albanese and Dr. Grant Erdmann of the AFRL in Brooks AFB, Texas, with whom we investigated (along with Dr. Devaney) fundamental questions on the practical launching of Kaiser's electromagnetic wavelets (see the paper [6]). The PI also collaborated with Dr. Hanoch Lev-Ari at the Department of Electrical and Computer Engineering at Northeastern University in a paper on intensity-only imaging [7].

### III. Publications Associated to this Project

In the past three years, most of the research of the PI's team has been based on this particular AFOSR grant, and this has been acknowledged in 14 refereed journal papers resulting directly or indirectly from this project, and in 22 conference papers and abstracts reporting our latest research in areas related directly or indirectly to the grant. Out of these publications, the following refereed journal papers stand as being the most directly related to the central goal of theoretically characterizing and exploiting radiation enhancement due to antenna substrates including metamaterials:

- A.J. Devaney, E.A. Marengo and M. Li, "Inverse source problem in non-homogeneous background media", *SIAM Journal on Applied Mathematics*, Vol. 67, p. 1353-1378, 2007, on the scalar version of the inverse source theory in non-homogeneous background media.
- E.A. Marengo, M.R. Khodja, and A. Boucherif, "Inverse source problem in non-homogeneous background media: Part II: Vector formulation and antenna substrate performance characterization", *SIAM Journal on Applied Mathematics*, Vol. 69, pp. 81-110, 2008, which generalizes the previous work to the full vector electromagnetic case including metamaterials and the important reactive power constraint.
- M.R. Khodja and E.A. Marengo, "Radiation enhancement due to metamaterial substrates from an inverse source theory", *Physical Review E*, Vol. 77, 046605, 13 pages, 2008, which investigates the important double-metamaterial-layer substrate design, in particular, the nested spheres configuration pairing positive and negative metamaterials.

- M.R. Khodja and E.A. Marengo, “Comparative study of radiation enhancement due to metamaterials”, Radio Science, Vol. 43, RS6S02, doi:10.1029/2007RS003803, 2008, which investigates the fair comparison of different substrates.

The following is the complete list of publications that resulted from this AFOSR project. For the refereed journal papers, the respective abstracts are provided for convenient reference.

### III.A. Refereed Journal Articles

1. M.R. Khodja and E.A. Marengo, “Comparative study of radiation enhancement due to metamaterials”, Radio Science, Vol. 43, RS6S02, doi:10.1029/2007RS003803, 2008.

**Abstract:** We present a comparative study on radiation enhancement due to metamaterial antenna substrates. The study is performed via electromagnetic inverse source theory and constrained optimization. The aim of the study is to understand the effect of reference antennas on enhancement level estimates. The geometry of the problem is that of a piecewise-constant, radially symmetric, three-region system (spherical core-shell system). Particular attention is given to the case when the core and the shell are made up of lossless materials having oppositely-signed constitutive parameters.

2. A.J. Devaney, G. Kaiser, E.A. Marengo, R. Albanese, and G. Erdmann, “The inverse source problem for wavelet fields”, IEEE Transactions on Antennas and Propagation, Vol. 56, p. 3179-3187, 2008.

**Abstract:** The theory of the *inverse source problem* is employed to compute a class of continuously distributed and compactly supported three-dimensional (volume) sources that radiate the scalar wavelets investigated by Kaiser as well as certain electromagnetic generalizations of these scalar fields. These efforts have shown that the scalar wavelet field can be radiated by a distributional source (generalized function) supported on a circular disk of radius  $a$  or an oblate spheroid surrounding that disk. Our main goal here is to replace this distributional source by a more conventional volume source that radiates the same

wavelet field outside its support volume. The equivalent volume sources computed in this paper are supported on (three-dimensional) spherical shells whose outer radius and inner radius are arbitrary. These sources are analytic functions of position within their support volumes for any finite, but arbitrarily large temporal frequency, and possess minimum L2 norm among all possible solutions to the inverse source problem with the given support volume constraint. Electromagnetic versions of the wavelet sources and fields are shown to be easily derived from their scalar wave counterparts.

3. E.A. Marengo, M.R. Khodja, and A. Boucherif, "Inverse source problem in non-homogeneous background media: Part II: Vector formulation and antenna substrate performance characterization", SIAM Journal on Applied Mathematics, Vol. 69, pp. 81-110, 2008.

**Abstract:** This paper solves analytically and illustrates numerically the full-vector, electromagnetic inverse source problem of synthesizing an unknown source embedded in a given substrate medium of volume  $V$  and radiating a prescribed exterior field. The derived formulation and results generalize previous work on the scalar version of the problem, especially the recent "Part I" paper [A.J. Devaney, E.A. Marengo, M. Li, Inverse source problem in non-homogeneous background media, SIAM J. Appl. Math. 67 (2007) pp.1353-1378]. Emphasis is put on substrates having constant constitutive properties within the source volume  $V$ , which, for formal tractability, is taken to be of spherical shape. The adopted approach is one of constrained optimization which also relies on spherical wavefunction theory. The derived theory and associated implications for antenna substrates are illustrated numerically.

4. M.R. Khodja and E.A. Marengo, "Radiation enhancement due to metamaterial substrates from an inverse source theory", Physical Review E, Vol. 77, 046605, 13 pages, 2008.

**Abstract:** In this paper the formalism of the electromagnetic inverse source theory is used to investigate radiation enhancement due to antenna substrates. Particular attention is given to sources that are confined within a spherical volume and are embedded within two nested

spheres of arbitrary materials. Emphasis is given to the special case when the two nested spheres are made up of materials with oppositely-signed constitutive parameters. The analysis comprises forward or radiation characterization for a given configuration as well as inverse-theoretic characterization. The forward characterization is focused on the singular value spectrum of the linear source-to-field mapping relevant to each configuration while the inverse-theoretic characterization is performed via the so-called “minimum-energy” sources capable of generating a prescribed exterior field. The derived formulation is based on constrained optimization and multipole theory. Importantly, it is non-antenna-specific. Thus this formulation enables fair comparison of different substrate configurations by comparing optimal radiation in each configuration (i.e., the “best” in each one), as governed by a formally tractable source-energy cost function that is physically motivated by ohmic loss control. The derived theory is accompanied by numerical results illustrating the effects on radiation enhancement of particular substrate designs.

5. J. C.-E. Sten and E.A. Marengo, “Inverse source problem in the spheroidal geometry: Vector formulation”, *IEEE Transactions on Antennas and Propagation*, Vol. 56, p. 961-969, 2008.

**Abstract:** A formulation based on Lagrangian optimization and spheroidal vector wave functions is presented for the vector electromagnetic inverse source problem of deducing a time-harmonic current distribution that is confined within a spheroidal volume, that generates a prescribed radiation field, and that is subject to given constraints on the source functional energy, which characterizes antenna current level, and the source’s reactive power, which models antenna resonance matching. The paper includes computer simulation results illustrating the derived inverse theory.

6. F. Simonetti, M. Fleming and E.A. Marengo, “An illustration of the role of multiple scattering in subwavelength imaging from far-field measurements”, *Journal of the Optical Society of America A*, Vol. 25, p. 292-303, 2008.

**Abstract:** Recently it has been proposed that the classical diffraction

limit could be overcome by taking into account multiple scattering effects to describe the interaction of a probing wave and the object to be imaged [Simonetti, Phys. Rev. E, 73, 036619, 2006]. Here this idea is illustrated by considering two point scatterers spaced much less than a wavelength apart. It is observed that while under the Born approximation the scattered field pattern is similar to that of a monopole source centered between the scatterers, multiple scattering leads to a more complicated pattern. This additional complexity carries information about the subwavelength structure and can lead to super resolution in the presence of large noise levels. Moreover, it is pointed out that the additional information due to multiple scattering is interpreted as a form of coherent noise by inversion algorithms based on the Born approximation.

7. E.A. Marengo, R.D. Hernandez and H. Lev-Ari, "Intensity-only signal-subspace-based imaging", Journal of the Optical Society of America A, Vol. 24, p. 3619-3635, 2007.

**Abstract:** A signal subspace method is derived for the localization and imaging of unknown scatterers using intensity-only wavefield data (lacking field phase information). The method is an extension of the time-reversal multiple signal classification imaging approach to intensity-only data. Importantly, the derived methodology works within exact scattering theory including multiple scattering.

8. E.A. Marengo, F.K. Gruber and F. Simonetti, "Time-reversal MUSIC imaging of extended targets", IEEE Transactions on Image Processing, Vol. 16, p. 1967-1984, 2007.

**Abstract:** This paper develops, within a general framework that is applicable to rather arbitrary electromagnetic and acoustic remote sensing systems, a theory of time-reversal Multiple Signal Classification (MUSIC)-based imaging of extended (non-point-like) scatterers (targets). The general analysis applies to arbitrary remote sensing geometry, and sheds light onto how the singular system of the scattering matrix relates to the geometrical and propagation characteristics of the entire transmitter-target-receiver system, and how to use this effect for imaging. All the developments are derived within exact scat-

tering theory which includes multiple scattering effects. The derived time-reversal MUSIC methods include both interior sampling as well as exterior sampling (enclosure) approaches. For presentation simplicity, particular attention is given to the time-harmonic case where the informational wave modes employed for target interrogation are purely spatial, but the corresponding generalization to broadband fields is also given. This paper includes computer simulations illustrating the derived theory and algorithms.

9. A.J. Devaney, E.A. Marengo and M. Li, "Inverse source problem in non-homogeneous background media", *SIAM Journal on Applied Mathematics*, Vol. 67, p. 1353-1378, 2007.

**Abstract:** The scalar wave inverse source problem (ISP) of determining an unknown radiating source from knowledge of the field it generates outside its region of localization is investigated for the case in which the source is embedded in a non-homogeneous medium with known index of refraction profile. It is shown that the solution to the ISP having minimum energy (so-called minimum energy source) can be obtained via a simple method of constrained optimization. This method is applied to the special case when the non-homogeneous background is spherically symmetric, and yields the minimum energy source in terms of a series of spherical harmonics and radial wave functions that are solutions to a Sturm-Liouville problem. The special case of a source embedded in a spherical region of constant index is treated in detail, and results from computer simulations are presented for this case.

10. J. C.-E. Sten and E.A. Marengo, "Transformation formulas for spherical and spheroidal multipole fields", *International Journal of Electronics and Communications (AEU)*, Vol. 61, p. 262-269, 2007.

**Abstract:** A transformation linking spherical multipole fields with generalized spheroidal multipole fields is derived. Applications including the inverse diffraction problem for spherically-scanned near field data for sources that due to conformal considerations are efficiently described using spheroidal (either oblate or prolate) volume support regions are discussed with the aid of numerical illustrations.

11. E.A. Marengo and F.K. Gruber, "Subspace-based localization and inverse scattering of multiply scattering point targets", EURASIP Journal on Advances in Signal Processing, Vol. 2007, Article ID 17342, 16 pages, 2007.

**Abstract:** The nonlinear inverse scattering problem of estimating the locations and scattering strengths or reflectivities of a number of small, point-like inhomogeneities (targets) to a known background medium from single-snapshot active wave sensor array data is investigated in connection with time-reversal multiple signal classification and an alternative signal subspace method which is based on search in high-dimensional parameter space and which is found to outperform the time-reversal approach in number of localizable targets and in estimation variance. A noniterative formula for the calculation of the target reflectivities is derived which completes the solution of the nonlinear inverse scattering problem for the general case when there is significant multiple scattering between the targets. The paper includes computer simulations illustrating the theory and methods discussed in the paper.

12. E.A. Marengo and F.K. Gruber, "Non-iterative analytical formula for inverse scattering of multiply scattering point targets", Journal of the Acoustical Society of America, Vol. 120, p. 3782-3788, 2006.

**Abstract:** This letter derives, in the exact framework of multiple scattering theory for point targets, a non-iterative analytical formula for the nonlinear inversion of the target scattering strengths from the scattering or response matrix that can be applied after the target positions have been estimated in a previous step via, e.g., time-reversal multiple signal classification (MUSIC) or other approach. The new formula provides a non-iterative analytical alternative to the iterative numerical solution approach for the same problem presented in a recent paper [A.J. Devaney, E.A. Marengo and F.K. Gruber, "Time-reversal-based imaging and inverse scattering of multiply scattering point targets", J. Acoust. Soc. Am., 118, 3129-3138 (2005)]. The two methods (non-iterative versus iterative) are comparatively investigated with two numerical examples.

13. J. C.-E. Sten and E.A. Marengo, "Inverse source problem in an oblate

spheroidal geometry", IEEE Transactions on Antennas and Propagation, Vol. 54, p. 3418-3428, 2006.

**Abstract:** The canonical inverse source problem of reconstructing an unknown source whose region of support is describable as a spheroidal (oblate or prolate) volume from knowledge of the far-field radiation pattern it generates is formulated and solved within the framework of the inhomogeneous scalar Helmholtz equation via a linear inversion framework in Hilbert spaces. Particular attention is paid to the analysis and computer illustration of flat, aperture-like sources whose support is approximated by an oblate spheroidal volume.

14. E.A. Marengo and M.R. Khodja, "Generalized power-spectrum Larmor formula for extended charged particle embedded in a harmonic oscillator", Physical Review E, Vol. 74, 036611, 11 pages, DOI: 10.1103, 2006.

**Abstract:** The nonrelativistic Larmor radiation formula, giving the power radiated by an accelerated charged point particle, is generalized for a spatially extended particle in the context of the classical charged harmonic oscillator. The particle is modeled as a spherically symmetric rigid charge distribution that possesses both translational and spinning degrees of freedom. The power spectrum obtained exhibits a structure that depends on the form factor of the particle, but reduces, in the limit of an infinitesimally small particle and for the charge distributions considered, to Larmor's familiar result. It is found that for finite-duration small-enough accelerations as well as perpetual uniform acceleration the power spectrum of the spatially extended particle reduces to that of a point particle. It is also found that when the acceleration is violent or the size parameter of the particle is very large compared to the wavelength of the emitted radiation the power spectrum is highly suppressed. Possible applications are discussed.

### III.B. Conference Papers and Presentations

- 1) E.A. Marengo, R.D. Hernandez, Y.R. Citron, F.K. Gruber, M. Zambrano, and H. Lev-Ari, "Compressive sensing for inverse scattering", XXIX URSI General Assembly, Chicago, Illinois, Aug. 7-16, 2008.

2) F.K. Gruber and E.A. Marengo, "Inverse scattering of extended targets by signal subspace approaches and the level set method", 2008 IEEE Geoscience and Remote Sensing Symposium, Boston, Massachusetts, July 7-11, 2008.

3) E.A. Marengo, "Compressive sensing and signal subspace methods for inverse scattering including multiple scattering", Invited talk at the Special Session on "Near sub-surface electromagnetic imaging: methods and applications", 2008 IEEE Geoscience and Remote Sensing Symposium, Boston, Massachusetts, July 7-11, 2008 (special session organized by Drs. Lorenzo Crocco and Amelie Litman).

4) E.A. Marengo, "Subspace and Bayesian compressive sensing methods in imaging", Invited talk at the Special Session on "Progress on Theory and Numerical Algorithms for Solving the Inverse Scattering Problems", of the Progress in Electromagnetics Research Symposium (PIERS) 2008, Cambridge, Massachusetts, July 2-6, 2008 (special session organized by Drs. Aria Abubakar and Dominique Lesselier).

5) E.A. Marengo and F.K. Gruber, "Inverse scattering by signal subspace and level set methods", Progress in Electromagnetics Research Symposium (PIERS) 2008, Cambridge, Massachusetts, July 2-6, 2008.

6) F.K. Gruber and E.A. Marengo, "Electromagnetic information theory for wireless and antenna systems", Progress in Electromagnetics Research Symposium (PIERS) 2008, Cambridge, Massachusetts, July 2-6, 2008.

7) E.A. Marengo, "Inverse scattering by compressive sensing and signal subspace methods", Invited talk at the Special Session on "Wave-based Signal Processing" of the IEEE Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP) 2007, St. Thomas, U.S. Virgin Islands, Dec. 12-14, 2007 (special session organized by Profs. Sven Nordebo and Mats Gustafsson).

8) E.A. Marengo, Invited talk: "Antennas and communication theory: Towards information-theoretic characterization of multi-configuration antenna systems", at "Military Antennas 2007" at Georgetown University Conference Center, Washington, D.C., September 26-28, 2007 (sponsored by the Institute for Defense and Government Advancement (IDGA)).

9) E.A. Marengo, "Further theoretical considerations for time-reversal MUSIC imaging of extended scatterers", Invited talk at the Special Session on "Time Reversal" of the IEEE Statistical Signal Processing Workshop 2007, pp. 304-306, Madison, Wisconsin, August 26-29, 2007 (ISBN:978-1-4244-1198-6) (special session organized by Profs. Jose Moura and Persefoni

Kyritsi).

10) E.A. Marengo, "Intensity-only MUSIC imaging", International Symposium on Electromagnetic Theory, URSI Commission B, Ottawa, Ontario, Canada, July 26-28, 2007.

11) E.A. Marengo, "Non-iterative imaging including multiple scattering", International Symposium on Electromagnetic Theory, URSI Commission B, Ottawa, Ontario, Canada, July 26-28, 2007.

12) E.A. Marengo and F.K. Gruber, "Electromagnetic information theory", International Symposium on Electromagnetic Theory, URSI Commission B, Ottawa, Ontario, Canada, July 26-28, 2007.

13) E.A. Marengo and M.R. Khodja, "Source synthesis in substrate media: Fundamental bounds", International Symposium on Electromagnetic Theory, URSI Commission B, Ottawa, Ontario, Canada, July 26-28, 2007.

14) J.C.-E. Sten and E.A. Marengo, "Electromagnetic inverse source problem in the spheroidal geometry", International Symposium on Electromagnetic Theory, URSI Commission B, Ottawa, Ontario, Canada, July 26-28, 2007.

15) E.A. Marengo, "Non-iterative inverse scattering including multiple scattering", URSI/CNC/USNC North American Radio Science Conference, Ottawa, Ontario, Canada, July 22-26, 2007.

16) E.A. Marengo and R.D. Hernandez, "Intensity-only localization and inverse scattering", URSI/CNC/USNC North American Radio Science Conference, Ottawa, Ontario, Canada, July 22-26, 2007.

17) E.A. Marengo and M.R. Khodja, "Source inversion in metamaterial substrate media: Fundamental limits, and applications", URSI/CNC/USNC North American Radio Science Conference, Ottawa, Ontario, Canada, July 22-26, 2007.

18) E.A. Marengo and F.K. Gruber, "Signal subspace-based non-iterative exact inverse scattering of point targets", Proc. of the International URSI/IEEE APS Annual Meeting, pp. 1383-1386, Albuquerque, New Mexico, July 2006.

19) E.A. Marengo and F.K. Gruber, "Time-reversal MUSIC imaging of extended scatterers, and a non-iterative exact inverse scattering formula", Proc. of the International URSI/IEEE APS Annual Meeting, p. 100, Albuquerque, New Mexico, July 2006.

20) E.A. Marengo and F.K. Gruber, "Time-reversal-based non-iterative exact inverse scattering of multiply scattering point targets", J. Acoust. Soc. Am., Vol. 119, p. 3247, 2006.

21) E.A. Marengo, "Time-reversal strategies for extended target focusing

and imaging, and clutter nulling”, at a Special Session on “Random media and rough surface scattering” organized by Dr. Saba Mudaliar, PIERS in Cambridge, p. 99 of the Conference Proceedings, Cambridge, Massachusetts, March 26-29, 2006.

22) E.A. Marengo and F.K. Gruber, “Time-reversal and signal subspace methods for imaging and inverse scattering of multiply scattering targets”, Proceedings of PIERS in Cambridge, p. 51, Cambridge, Massachusetts, March 26-29, 2006.

## IV. Main Accomplishments and Scientific Merit

This AFOSR effort consisted of the development of a new inverse source theory for sources embedded in non-homogeneous backgrounds, with a focus on case studies in antenna characterization and synthesis including the rigorous evaluation of fundamental antenna performance bounds applicable to antenna-embedding substrates including metamaterials [8,9]. Motivation was provided by the possibility of embedding antennas in designer background media so as to obtain radiation performance that would not be possible for comparable antennas in free space. Of much interest among such media are metamaterials which have been the focus of a lot of attention in the present decade in connection with several effects such as the “perfect lens” [10], cloaking devices for invisibility [11], and highly directive radiating devices [12,13], among other. Of particular interest to the present effort was the latter effect of highly directive radiating devices, studied by other groups from the experimental and more practical perspective, and *for particular antennas*.

But to fairly assess and compare radiation enhancements due to substrates one must consider benchmarks that are *intrinsic* to the substrate, i.e., that are *device-independent*. For example, in information theory one talks about the information capacity of a given communication channel, and this capacity (bits/second) is a fundamental performance bound that is intrinsic to the channel, i.e., it is not tied to a particular communication algorithm; also, in estimation theory one talks about the fundamental Cramer-Rao bound for the error of an unbiased estimator, and this holds for any unbiased estimator, i.e., it is independent of the particular estimation algorithm. The interest in this project of formulating an inverse-source-theoretic approach to characterizing antennas in substrates was, similarly, motivated by the need to develop a solid framework to characterize the *intrinsic* capabil-

ities of given antenna substrates, under given constraints modeling physical limitations or resources at one's availability. From such characterization, one can then also compare different antenna substrates and configurations, and also gain understanding on the physical mechanisms giving rise to given enhancements. Thus the particular approach that was explored in detail in this work was the theoretical characterization of the enhancement in antenna performance (e.g., antenna directivity, antenna radiated power, and -due to reciprocity- also the associated reception capabilities) via antenna substrates functioning under given physical constraints (antenna dimensions, permissible currents, input power, etc.), for general antennas viewed as primary sources operating under such constraints.

The key contributions have been reported in the papers [14–17]. In those studies, the inverse source problem in non-homogeneous and metamaterial background media was formulated using solution constraints modeling resources and constraints relevant to practical antennas. The developed inverse source theory has been based on optimization theory and multipole theory. The inverse source problem for a source of known support  $V$  consists of the estimation of the source from knowledge of the field it generates, under given boundary conditions (in particular, the radiation condition) everywhere outside  $V$ . Without solution constraints the inverse source problem exhibits nonuniqueness due to the possible existence within the source support of nonradiating sources whose generated fields vanish identically outside the source region. But the selection of certain solution constraints renders the inverse source problem unique. In the inverse source theory in non-homogeneous and metamaterial background media developed in this project particular attention was given to the familiar constraint of minimizing the source's L2 norm or functional energy (which essentially restricts the antenna "current levels"). But from that basic constraint as starting point, which to a large extent already characterized the essence of radiation enhancements due to substrates (as we showed with the scalar theory [14]), the PI's group was able to subsequently generalize to more general constraints such as the minimizing of the source functional energy for a prescribed (say, zero) reactive power at antenna terminals, and the minimizing of the reactive power for a given permissible source energy level, among others [15–17]. In the key papers [14–17], detailed analytical and computational studies have also been reported which compare free space versus substrate configurations and which clearly demonstrate the enhancement due to substrates in the sense that one can achieve, e.g., a given antenna directivity with smaller antennas,

or in general with less resources such as current levels, relative to comparable antennas in free space.

The research program followed a logical path. First it investigated the scalar case in the framework of the Helmholtz operator [14]. The results in that paper were later generalized to the full vector case in the comprehensive Part II paper [16] which, besides including the full vector formalism, also incorporated both the minimum energy source constraint in [14] and the constraint of tuning the antenna with the help of the substrate so that its reactive power is zero. Finally, the contribution [15] elaborated on the special case of the two-nested spheres configuration which we found to yield quite high radiation enhancements, and the paper [17] carried out a detailed analysis of the issue of fairness in comparing the performance of the same antenna, or more generally of different antennas, in different substrates.

#### **IV.A. Scientific Merit**

It is important to mention that through the present AFOSR the PI's team expanded significantly the body of literature on the full vector version of the inverse source problem not only in the aforementioned papers [15–17] but also in other papers on the inverse problem for wavelets [6] and the inverse problem for spheroidal antennas [3–5].

Furthermore, previous to this effort, and even within the scalar version of the theory, there had been very few treatments of the inverse source problem for non-homogeneous background media. The only other treatments on this problem that were available in the literature were Refs. [18, 19]. Both of these papers consider only the scalar case. The theory developed in this project expanded significantly both in technique and in scientific scope (antenna theory) the pioneering work in these papers which considered only the scalar case and did not provide illustrations of the general theory. Also, unlike in past work, the new theory of this project applies to the modern metamaterials.

### **V. Broader Impacts**

The most obvious broader impact of the developed inverse source theory for antennas embedded in substrates is its impact in more general imaging applications. For example, in certain applications one can use embedding

media to enhance imaging resolution, and this holds in both passive and active (inverse-scattering-based) imaging.

In addition, even though the focus of this work was on developing a new inverse source theory in non-homogeneous and metamaterial background media motivated by antenna applications, the PI's team also worked on other scientific problems. Of particular interest was the work on non-iterative signal-subspace-based imaging methods for inverse scattering including multiple scattering.

The main additional contributions which complement the PI's team work on inverse antenna theory in non-homogeneous and metamaterial backgrounds are:

- The development of a new intensity-only signal-subspace-based imaging methodology that allows non-iterative imaging of point and extended scatterers, within exact scattering theory including multiple scattering (the key paper reporting this work is [7]).
- The development of new signal-subspace-based imaging algorithms for extended scatterers that apply in both narrowband and broadband regimes, general remote sensing apertures, and both scalar and vector fields. This contribution is reported in [2]. The methods proposed in [2] are many, and include both interior sampling and exterior sampling (enclosure) approaches. Importantly, they apply in the general case including multiple scattering.
- The development of a detailed inverse source theory for spheroidal antennas, which has the appeal of conformally modeling antennas of various shapes (e.g., planar antennas via oblates, and elongated antennas via prolates). This work was reported in three papers [3–5].
- The investigation in [1] of the role of multiple scattering in super-resolution imaging. This work explores canonical systems of multiply scattering point targets and shows that multiple scattering can be an asset in active imaging since the scattering process including multiple scattering can communicate evanescent near field information about the scatterer's position into the far zone, thereby enabling the inversion of subwavelength features not possible with weak scattering (Born approximation).

This grant also stimulated interactions with the inverse antenna theory program of Dr. Gerald Kaiser at the Center for Signals and Systems in Austin, Texas, and with Dr. Richard Albanese and Dr. Grant Erdmann from the AFRL in Texas, with whom Dr. Anthony Devaney and the PI in this effort collaborated (as a five-member team) toward the important goal of learning how to practically launch Kaiser's electromagnetic wavelets. The paper [6] summarizes the research from this effort.

Finally, the PI has presented the results developed during this grant period in several conference talks (a total of 22 conference papers and associated presentations), and in several invited talks at universities in the USA and abroad (MIT, University of Delaware, Imperial College London, Lund University, Second University of Naples, and so on) and at companies and research laboratories (Raytheon Integrated Defense Systems, Sudbury, MA, AFRL in Rome, NY, and so on).

## VI. Conclusions and Recommendations

This AFOSR project contributed to the development of a new theory of the full vector, electromagnetic inverse source problem which is applicable to sources embedded in substrates, with applications to the analysis, source-synthesis and characterization of antennas embedded in substrates. The developments in [14–17] emphasized sources in a spherical volume which facilitated analytical treatment via spherical wave multipoles but the general concepts hold for more general cases.

The central goal of this project was to gain understanding from first principles of potential radiation enhancements (reduction of required antenna resources (physical size, current levels, level of tuning, and so on) for a given far field) due to such substrates. This problem was treated in the present work within a general and non-device-specific framework whose predictions (such as performance bounds) under normalized resources are fundamental.

The results were discussed addressing separately the cases of small versus large or resonant antennas, with the overall conclusion that for small antennas one can significantly enhance the radiated power or compress source size via the substrates under normalized antenna resources, while for larger antennas the use of substrates can significantly enhance both radiated power and directivity (related to the number of essentially independent field modes that can be radiated effectively) under the given resources. This investigation has

confirmed from a fundamental electromagnetic and inverse theoretic point of view that the metamaterials can enhance radiation significantly, mostly when they are paired in double-negative and double-positive pairs such as in the nested shell configuration [15, 17].

Some future research directions are recommended next. The present theory holds for a given frequency. Since physically the enhancements due to antenna substrates are due to resonances, one naturally wonders whether they will find practical applicability over broad bands. This is one of the main challenges of antenna and antenna substrate designers, and it naturally calls for the development of an associated broadband inverse theory for background media. Thus extension of the inverse source theory to the broadband regime and dispersive embedding media is an important natural continuation of the lines of research developed in the present research program. It is also worth noting that in addition to metamaterial substrates, one could also employ much simpler and cheaper, and yet quite effective antenna substrate designs in certain applications. A concrete example is the subwavelength antenna array design in [20] where coupling (multiple scattering) between the antenna elements is exploited rather than avoided, with encouraging subwavelength time-reversal communications [20]. The particular research avenue of exploring and exploiting further such rather simple coupling-inducing antenna arrays deserves further attention.

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